Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Y.M. Rangkuti *et al* 2022 *J. Phys.: Conf. Ser.* **2193** 012091

Control optimal analysis of SEIR model of covid 19 spread in Indonesia

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Abstract. In this paper we use optimal control techniques, SEIR model of Covid 19 in case Indonesia, in order to establish vaccination, personal protective measures and treatment plans to control the spread of an infectious disease. We validate them by the use of the Maximum Principle. The findings revealed that the best practice of combining all three intervention measures considerably reduces the proportion of people who are exposed and symptomatic, as well as those who are asymptomatic.

1. Inroduction

Within a population, contagious infectious diseases can be a significant societal issue. It can result in tragic deaths, a loss of quality of life, and a reduction in a population's active life, all of which have economic and societal ramifications. In order to prevent the spread of infectious diseases, vaccination and treatment strategies must be devised. These policies must consider the social and economic consequences of existing sick people, as well as the costs of vaccination and treatment, which include both the manufacturing and administration of vaccines and medicines. SEIR models are often used to estimate the spread of infectious illness within a population (e.g., Wintachai and Prathom (2021) [1], Annas et al. (2020) [2], Wei and Xue (2020) [3], and Das (2021) [4]. These are compartmental models, in which the population is separated into groups based on the stage of the infectious disease in their bodies. In recent years, SEIR models combined with optimal control techniques have proven to be an effective tool in controlling the spread of infectious diseases (Pinho and Nogueira (2017) [5], Fatima et al. (2021) [6], Deressa and Duressa (2021) [7]).

In this study, we also integrate SEIR models to optimal control techniques to develop vaccination and treatment programs to minimize an infectious disease from spreading. It's worth noting that, despite the fact that we're working with a fictitious population and a COVID-19 infectious disease, the results may be utilized to prevent the spread of a real infectious disease among actual people. The same normalized SEIR model is used as de Pinho and Nogueira (2017) [8]. Normalization offers the advantage of covering populations of various sizes in a single problem and reducing model complexity by permitting the deletion of one differential equation and one parameter (death rate). Instead of a single controller, as in de Pinho and Nogueira, we consider two controllers (2017) [5]. We also add a new compartment to the model that is relevant to infected persons who are being treated. A cost function based on $\boldsymbol{L^2}$-type objectives are developed and utilized to three scenarios. The Maximum Principle is used to the considered situation as we answer our optimal control problem analytically.

The following is a breakdown of how this paper is organized. We provide an optimal control problem for SEIR models with $\boldsymbol{L^2}$ cost in Section 2. The Maximum Principle is applied to the optimal issue in Section 3 are presented. The maximum principle is used to partially validate the generated solutions. The latter section of this paper contains the conclusions).

2. SEIR Model of COVID 19 Disease

The well-known SEIR compartmental model is taken into consideration to characterize the transmission of the disease in a given population. In general, $N\,(t)$ represents the total population at time t. $N\,(t)$ is then separated into four compartments: S, E, I, and R. $S(t)$ represents the population susceptible to the disease, $E(t)$ represents the number of people who have been exposed to the disease but are not yet infectious, and $I(t)$ represents the number of people who have been infected with the disease and may infect those who are susceptible through direct transmission, and $R(t)$ signifies those who are disease-free. Annas et al. in 2020 [2] have been constructed the SEIR model by considering vaccination and isolation factors as model parameters. The model was written as the following controlled dynamical system:

$$\dot{S} = \mu N - (\alpha I + \mu + v)S, \tag{1}$$

$$\dot{E} = \alpha IS - (\beta + \mu)E, \tag{2}$$

$$\dot{I} = \beta E - (\sigma + \delta + \mu)I, \tag{3}$$

$$\dot{R} = \delta I + \rho S - \mu R, \tag{4}$$

Here, nonnegative initial conditions S_0, E_0, I_0, R_0, M_0, and N_0 are used. $N\,(t) = S(t) + E(t) + I(t) + M\,(t) + R\,(t)$.

The model in Eqs. (1)-(4) has been analysed, Annas et al. [2] found the equilibrium points in case free disease and endemic are $E_0 = (S, E, I, R) = \left(\frac{\mu}{(\mu+v)}, 0, 0, \frac{v}{(\mu+v)}\right)$ *and* $E_0 = (S^*, E^*, I^*, R^*) = \begin{pmatrix} \frac{(\mu_i+\delta+\mu)(\beta+\mu)}{\alpha\beta}, \frac{\alpha\beta\mu-(\mu_i+\delta+\mu)(\mu+v)}{\alpha\beta}, \\ \frac{\alpha\beta\mu-(\mu_i+\delta+\mu)(\mu+v)(\beta+\mu)}{\alpha(\mu_i+\delta+\mu)(\beta+\mu)}, \\ \frac{\delta\alpha\beta^2\mu-\beta(\mu_i+\delta+\mu)(\mu+v)(\beta+\mu)-v\left((\mu_i+\delta+\mu)(\beta+\mu)\right)^2}{\beta\alpha^2(\mu_i+\delta+\mu)(\beta+\mu)} \end{pmatrix}$. From the result, Indonesia is endemic COVID-19 if no vaccine, thus the simulation results showed that the vaccine can accelerate COVID-19 healing and maximum isolation can slow the spread of COVID-19A.

3. Optimal Control

Annas et al. [2] stated that the basic reproduction number R_0 for the endemic case of Covid-19 with only 1% vaccination is $R_0 = 3.2094$. This means that, if a person is infected with Covid-19 it will infect 3 other people. Whereas the R_0 value for simulations explained that 50% vaccine would reduce transmission of COVID-19 and 100% did not cause spreading of Covid-19 in Indonesia. From Annas et al. 's report, we believe that a vaccination exists to protect those who are susceptible to the disease from becoming infected. We also know that immunizations aren't 100% effective. This indicates that if we vaccine $u_1(t)S(t)$ and a personal protective measure such as facemasks, regular hand washing, and social distancing people $u_2(t)S(t)$ at time t, only $u_1(t)S(t)$ people will become immune, or, to put it another way, $(u_1(t)+u_2(t))S(t)$ people and also $u_2(t)S(t)$ will shift to compartment R. Here $u_1(t)$ represents the vaccination rate at t. Optimal control issues incorporating SEIR models are considered to develop vaccination techniques to control the spread of illnesses into a certain population in a set time period (Neilan and Lenhart (2010) [10] and Biswas et al. (2014)[11]). Here, we assume that by vaccination the susceptible and providing medical care to those who are infected, we can prevent the spread of a horizontally transmitted disease. We expect that an infectious person who is receiving treatment will not infect others. Treatment, however, takes time, as we all know.

Control strategies are crucial in preventing the spread of COVID-19. In this study, three intervention options to prevent the pandemic's transmission that are time dependent control variables, notably: u_1 denotes a COVID-19 control technique involving vaccine and u_2 is a control method that involves the use of personal protective measures such as facemasks, regular hand washing, and social distancing, all of which are regarded effective in the Indonesian environment, and u_3 refers to COVID-19 patients who are being treated to lessen the severity of their sickness. The state equation is obtained by adding these control variables into (1).

$$\dot{S}=\mu N-(\alpha I+\mu+v)S+(u_1(t)+u_2(t))S(t),$$

$$\dot{E}=\alpha IS-(\beta+\mu)E,$$

$$\dot{I}=\beta E-(\sigma+\delta+\mu+u_3)I,$$

$$\dot{R}=(\delta+u_3)I+(\rho(u_1(t)+u_2(t))S-\mu R, \tag{5}$$

The goal now is to establish an optimal control for the preventive strategies/ measures u_2, u_2 and u_3 for the comparatively lowered coastline of the preventative techniques. The following control constraints are used throughout the paper:

$$0\le u_i(t)\le 1, i=1,2,3, a.e. t\in[0,T],$$

where the parameter T is the time horizon.

For optimal control to occur, the pontryagin's maximal principle [12] establishes the necessary and sufficient conditions The function that minimizes the number of susceptible case S, Exposed cases E, number of symptomatically infected cases I, number of asymptomatically infected cases A over a time interval of [0, T] can be defined as

$$J_{op}(u_i,\Omega)=\int_0^T\left(c_0E+c_1I(s)+\sum_{i=1}^{3}\frac{b_i}{2}u_i^2\right)dt, \tag{6}$$

where Ω denotes biological feasible region defined in ref. [2], c_0 are positive weight to balance the factors and b_1, b_2, b_3 measures the relative cost of intervention strategies over the time range of $[0,T]$. Minimizing equation (6) provides an optimal control $u_i^*, i=1,2,3$ such that

$$Jop(u_1^*,u_2^*,u_3^*)=\min_{u\in U} Jop(u_1(t),u_2(t),u_3(t)), \tag{7}$$

where the control set is given by $U=\{u_i: 0\le u_i(t)\le 1, 0\le u_i(t)\le 1, t\in[0,T]\}$ subjected to the constraint given by system of differential equation (5). The necessary conditions that need to be satisfied by optimal control called Pontryagin's Maximum Principle converts equation (6) and (5) into a problem of minimizing point-wise a Hamiltonian Hs with respect to $u_i(t)$, where Hs is defined as,

$$\begin{aligned} Hs(y,u_i(t),u_i(t),u_i(t),\lambda_S,\lambda_E,\lambda_I,\lambda_R,t) &= c_0\mathrm{E}(t)+c_1\mathrm{I}(t)+\left(\frac{b_1}{2}*u_1^2+\frac{b_2}{2}*u_2^2+\frac{b_3}{2}*u_3^2\right)(t)+\lambda_S h_S+\lambda_E h_E \\ &+\lambda_I h_I+\lambda_R h_R, \end{aligned} \tag{8}$$

where

$$h_1=\mu(\mathrm{S(t)}+\mathrm{E(t)}+\mathrm{I(t)}+\mathrm{R(t)})-(\alpha\mathrm{I(t)}+\mu+\nu)\mathrm{S(t)}-(u_1+u_2)\mathrm{S(t)},$$

$$h_2 = \alpha \mathrm{I(t)S(t)} - (\beta + \mu)\mathrm{E(t)},$$
$$h_3 = \beta \mathrm{I(t)} - (\sigma + \delta + \mu)\mathrm{I(t)} - u_3\mathrm{I(t)},$$
$$h_4 = (\delta + u_3)\mathrm{I(t)} + (v + u_1 + u_2)\mathrm{S(t)} - \mu \mathrm{R(t)},$$

Differentiating the Hamiltonian function with respect to the compartment variables gives the adjoint variables $\lambda_j, j \in \{SEIR\}$ corresponding to the system given as follows:

$$\dot{\lambda}_S = (\alpha \mathrm{I(t)} + v + u_1 + u_2)\lambda_S - \mathrm{I(t)}\lambda_E \alpha - (\mathrm{R(t)}v + u_1 + u_2)\lambda_R, \tag{9}$$
$$\dot{\lambda}_E = -\mu\lambda_S + (\beta + \mu)\lambda_E - c_0, \tag{10}$$
$$\dot{\lambda}_I = (\mathrm{S(t)}\alpha - \mu)\lambda_S - \alpha \mathrm{S(t)}\lambda_E + (\delta - \beta + \mu + \sigma + u_3)\lambda_I - (u_3 + \delta)\lambda_R - c_1, \tag{11}$$
$$\dot{\lambda}_R = \mu(-\lambda_S + \lambda_R), \tag{12}$$

and $\lambda_S, \lambda_E, \lambda_I, \lambda_R$ are the adjoint variables, $\lambda_i = (\lambda_S, \lambda_E, \lambda_I, \lambda_R), y \in \{S, E, I, R\}$. Now, setting the transversality condition

$$\lambda_i(T) = 0, j \in \{\lambda_S, \lambda_E, \lambda_I, \lambda_R\}. \tag{13}$$

We obtain the optimal controls and the optimality conditions respectively as

$$u_1 = \frac{\mathrm{S(t)}\lambda_S - \mathrm{S(t)}\lambda_R}{b_1},$$
$$u_2 = \frac{\mathrm{S(t)}\lambda_S - \mathrm{S(t)}\lambda_R}{b_2},$$
$$u_3 = \frac{\mathrm{I(t)}\lambda_I - \mathrm{I(t)}\lambda_R}{b_3}, \tag{14}$$

with the stationer condition

$$u_1^*(t) = \min\left[\max\left[\frac{\mathrm{S(t)}\lambda_S - \mathrm{S(t)}\lambda_R}{b_1}\right], 1\right],$$
$$u_2^*(t) = \min\left[\max\left[\frac{\mathrm{S(t)}\lambda_S - \mathrm{S(t)}\lambda_R}{b_2}\right], 1\right],$$
$$u_3^*(t) = \min\left[\max\left[\frac{\mathrm{I(t)}\lambda_I - \mathrm{I(t)}\lambda_R}{b_3}\right], 1\right], \tag{15}$$

Note that, state equation (6), the adjoint equation (8) together with the characterization of the optimal control (15) and the transversality condition (13) are said to be Optimality system. From (15), $\lambda_S < 1 + \lambda_R$ and $\lambda_I < 1 + \lambda_R$ for $b_1, b_2, b_3 > 0$.

4. Conclusions

In the present work, we have discussed optimal control of SEIR model of Covid 19 disease which was constructed by Annas et al. [12]. An optimal analysis of the model for the purpose of assessing the effect of vaccination, effect of personal protective measures and effect of treating hospitalized or isolated cases in mitigating transmission of COVID-19 was conducted. The findings revealed that the best practice of combining all three intervention measures considerably reduces the proportion of people who are infected and symptomatic, as well as those who are asymptomatic.

References

[1] Wintachai, P. & Prathom, K. 2021. Stability analysis of SEIR model related to efficiency of vaccines for COVID-19 situation, *Heliyon*, 7 (2021) e06812

[2] Annas, S., Pratama, M.I., Rifandi, M., Sanusi, W., & Side, S. 2020. Stability analysis and numerical simulation of SEIR model for pandemic COVID-19 spread in Indonesia, *Chaos, Solitons and Fractals*, 139, 110072

[3] Wei F. & Xue, R. 2020. Stability and extinction of SEIR epidemic models with generalized nonlinear incidence, *Mathematics and Computers in Simulation*, 170, 1-15

[4] Das, K., Murthy, B.S.N., Samad, S.A., & Biswas, M.H.A. 2021. Mathematical Transmission Analysis of SEIR Tuberculosis Disease Model, *Sensors International*,

[5] De Pinho, M.D. R., & Nogueira, F.N., 2018 Costs analysis for the application of optimal control

to SEIR normalized models, IFAC PapersOnLine, 51 (27),122–127
[6] Fatima, B., Zaman, G., Alqudah, M.A. & Abdeljawad, T. 2021. Modeling the pandemic trend of 2019 Coronavirus with optimal control, *Analysis, Results in Physics*, 20, 103660
[7] Deressa, C.T. & Duressa, G.F., 2021, Modeling and optimal control analysis of transmission dynamics of COVID-19: The case of Ethiopia, *Alexandria Engineering Journal* (2021) 60, 719–732
[8] De Pinho, M.D.R. and Nogueira, F.N. (2017). On application of optimal control to seir normalized models: pros and cons. *Mathematical Biosciences and Engineering (MBE)*, 14(1), 111–126.
[9] Okosun, K.O., Rachid, O., and Marcus, N. 2013. Optimal control strategies and cost-effectiveness analysis ofa malaria model. *Biosystems*, 111(2), 83–101.
[10] Neilan, R.M. and Lenhart, S. (2010). An introduction to optimal control with an application in disease modeling. *DIMACS Series in Discrete Mathematics*, 75, 67–81.
[11] Biswas, M.H.A., Paiva, L.T., and de Pinho, M.D.R. 2014. A SEIR model for control of infectious diseases with constraints. *Mathematical Biosciences and Engineering*, 11(4), 761–784.
[12] Pontryagin, L.S., 1997, Mathematical theory of optimal processes. Routledge; 2018 May 3.Sanchez MA, Blower SM. Uncertainty and sensitivity analysis of the basic reproductive rate: tuberculosis as an example, *American journal of epidemiology*. 145(12), 1127-37.

Acknowledgment
Authors wishing to acknowledge assistance or encouragement from Universitas Negeri Medan with Project’s number 0012/UN33.8/PL-PNBP/2021