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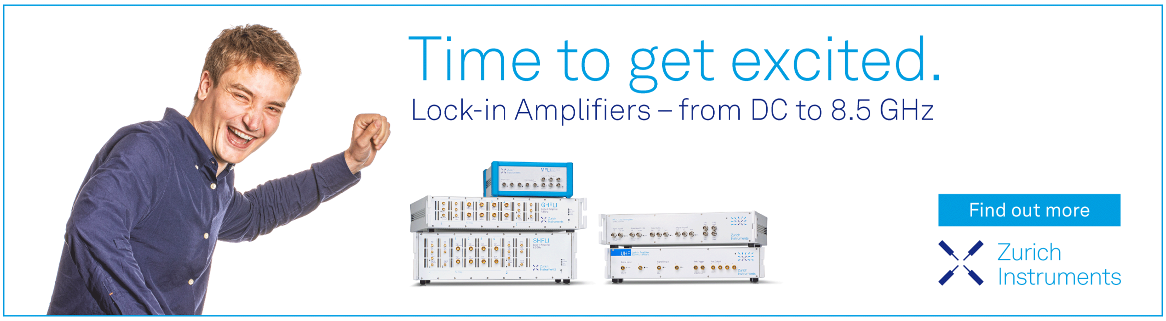

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Improvement Habits of Minds in Constructing Mathematical Proof Using DNR-Model

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Abstract. This study aims to improve students' mind habits in constructing mathematical proof through DNR-Model. DNR-Model consists of three main stages: Duality (how to understand and how to think), necessity, and repeated reasoning. This research is a quantitative study with a quasi-experimental design. The samples were then second-year students of the mathematics education department; the Muhammadiyah University of Tangerang consists of 60 students who were divided into two class groups. Two classes were selected as an experimental class that provides learning models for DNR-Model, and one other class as a control class that subjects are given conventional learning. The results showed that there aren't differences in the increase in the habits of minds of students in constructing mathematical proof based on MPK (Mathematical Prior Knowledge) levels of both high, medium, low, and overall learning based on DNR-Model and conventional learning. The experimental group's average n-gain habits of mind are 0.27, or the lower category than the average n-gain of the control group is 0.28 or the low category. The low Habits of Mind of students has less influence on the ability to construct mathematical proofs.

INTRODUCTION

Students' thinking ability is inseparable from how students construct their knowledge and use their knowledge appropriately by the problems. The attitude of students as this will guide students toward habituation ineffective thinking. This habit of thinking is known as the habit of mind [1]. Students' attitudes during this learning process will have an impact on meaningful learning outcomes. Students don't just memorize or get grades. The student learning process should be directed through an effective process of thinking so that students can skillfully put themselves in a variety of situations by relying on thinking abilities. Students can establish good habits in the everyday learning process. Students may begin to behave productively to discipline and train intelligent students.

Habits of mind has very good benefits. The application of habits of mind will help students always to use their time productively and hone students' intelligence. Learning habits like this course are very needed by students both in their daily lives and at certain times, such as final examinations. Learning methods that are directed, organized, and appropriate to provide opportunities for students to gain meaningful knowledge. Costa and Kallick [1], which consists of 16 categories, have developed the habit of Mind habituation. The categories are persisting, managing impulsivity, striving for accuracy, thinking and communicating with clarity and precision, gathering data through all senses, questioning and posing problems, metacognition, listening with understanding and empathy, thinking flexibly, creating, imagining, innovating, finding humor, responding with wonderment and awareness, applying past knowledge to new situations, taking a responsible risk, thinking interdependently, and remaining open to continuous learning. The development of these 16 categories is expected to guide students to be intelligent students both in the classroom and in their environment.

In a college environment, one of the mathematical abilities' students must achieve mathematical proofing ability. The ability to prove is the highest level of mathematical ability at the level of advanced thinking [2]. The results of previous studies indicate that the ability to prove is a mathematical problem that is difficult to achieve by most students

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[3][2][4][5][6]. Then the results of research on mathematical proofing ability show that students' ability is still low [7] [8][9][10][11]. Many factors cause students difficulties in constructing a mathematical proof, including experience in proving. Experience in constructing mathematical proof is an important factor that causes students to fail in constructing a mathematical proof. Moore [3] stated that experience in constructing proof is an important factor in constructing a mathematical proof. Knowledge of proofing strategies is another factor that causes students to fail [12]. On the other hand, students also still lack the confidence to validate whether the proof is valid or not [13].

In proofing, in addition to solving problems, students are required to be able to prove logically and scientifically reasoned. Therefore, need for habituation in thinking of constructing a mathematical proof. The habit of mind in solving problems, especially in constructing a mathematical proof, is important for students. With their habits of mind, students are expected to be able to think mathematically precise proof strategies, link relationships between mathematical concepts, and write mathematical proofs correctly and adequately. To support an active thinking process, a learning model that facilitates and provides space for students to think and understand in solving mathematical proof problems is needed. Efforts to improve creative thinking should be made through teaching in the classroom [14]. One of the models relevant to the habit of constructing a mathematical proof is a DNR-Model [5].

DNR stands for Duality, necessity, and repeated reasoning was taken from three pedagogical principles of DNR-Model, namely Duality principle, Necessity Principle, and Repeated Reasoning Principle [15]. DNR has been developed in a series of teaching experiments, has been studied for almost three decades in elementary, secondary, and college mathematics, as well as teaching experiments in the context of professional development for teachers at every level of education [16].

Understanding how to understand and how to think has a technical meaning in DNR are two different knowledge—understanding the reference to the products, such as definitions, conjectures, theorems, proofs, problems, and solutions. At the same time, thinking refers to the practice of mathematics used to produce the products. Examples of ways of thinking include empirical reasoning, deductive reasoning, structural reasoning, heuristic, and beliefs about the nature of mathematical knowledge and the acquisition process. One of the most important pedagogical implications of learning is the necessity principle, the second basic principle of the DNR-Model.

Harel [17] defines necessity as the need for students to learn what the teacher taught them. They should have such needs, which "needs" refers to intellectual needs. Harel [17] categorizes intellectual needs into five: Need for Certainty, Need for Causality, Need for Computation, Need for communications, and Need for Structure.

Then, repeated reasoning, which intentionally exercises to be repeated, is an important factor in cognitive processes. Repeated reasoning drills and practices routine problems and is very important for the internalization process, which is a conceptual state in which a person can apply knowledge independently and spontaneously and reorganize knowledge.

METHOD

This research is a quantitative study with a quasi-experimental design. The sample in this study was the second-year student majoring in mathematics education, Muhammadiyah University of Tangerang being supported discrete mathematics. The total number of samples is 60 students, divided into two groups of classes. The first group as an experimental class consisted of 28 students, and the subject gained learning using the DNR-Model. And the other class, as a class as control class, consists of 32 students, and the subjects were given learning with conventional models. In this study, the dependent variable is DNR-Model and conventional learning. The independent variable is the habits of mind of students in constructing a mathematical proof. And the control variables are prior mathematical knowledge (MPK) divided into high, medium, and low levels. There are 16 indicators of students' habits of mind in constructing mathematical evidence, namely, persisting, managing impulsivity, listening and understanding and empathy, thinking flexibility, metacognition, striving for accuracy, questioning, and problem-posing, applying past knowledge to the new situation, thinking and communicating with clarity and precision, creating, imagining and innovating, responding with wonderment and awe, Taking the responsible risk, Finding humor, Thinking interdependently, and Reaming open to continuous learning [1]. Habits of mind of students are measured through a questionnaire that was adapted from the Habits of Mind's Costa and Kallick. This questionnaire consists of 39 items statements regarding the habits of mind of students in constructing a mathematical proof. The instrument has been tested for validity by five validators and reliability testing of 0.889 in the high category. Pre-respond and post-respond data are analyses with t-test, t'-test, and two-way ANOVA test.

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RESULT AND DISCUSSION

Improved habits of mind students in constructing a mathematical proof are obtained from the n-gain based on pre-respond and post-student responses to a questionnaire given to both classes. The experimental class obtains DNR-Model, while the control class receives conventional learning treatment. The distribution of n-gain habit of mind data for the two classes is shown in the steam-leaf diagram in Table 1.

TABLE 1. Distribution of n-gain habits of mind experimental class and control class.

DNR-Model		Steam	Conventional	
f	Leaf		Leaf	f
2	9 8	0	-	
2	4 1	1	2 3	2
3	8 7 6	1	5 7 7 9 9	5
8	4 4 3 2 2 1 1 1	2	0 1 2 2 4 4 4	7
9	9 9 8 7 7 7 6 5 5	2	6 6 6 6 6 7 7 8 8 8	10
2	2 1	3	0 0 1 1 2 2 3 4	8
2	8 6	3	-	

Steam unit: 0.10

TABLE 2. Descriptive statistics n-gain habits of mind experimental classes and control classes based on the level of MPK.

MPK	Statistics	n-gain		Total
		DNR-M	Conventional	
High	n	6	5	11
	$\bar{x}$	0.20	0.19	0.20
	s	0.11	0.16	0.12
	Category	Low	Low	Low
Medium	n	18	20	38
	$\bar{x}$	0.28	0.30	0.29
	s	0.08	0.09	0.09
	Category	Low	Low	Low
Low	n	4	7	11
	$\bar{x}$	0.33	0.32	0.32
	s	0.03	0.04	0.04
	Category	Medium	Medium	Medium
Total	n	28	32	60
	$\bar{x}$	0.27	0.28	0.28
	s	0.09	0.10	0.10
	Category	Low	Low	Low

Based on Table 1 shows that there are differences in the character of the increase in student habits of mind in constructing the mathematical proof between the experimental class and control class. The N-gain data of the experimental class is cantered in the middle of the index interval 0.21 to 0.29, while the n-gain data of the control class is more inclined to the upward and cantered at the index interval 0.20 to 0.34. Overall, both the experimental class and a control class increased the habits of mind students in constructing a mathematical proof. Furthermore, it needs to be reviewed further based on descriptive statistics, normality tests, and homogeneity of the two groups to see its significance. Table 2 presents the descriptive statistics of data N-gain habits of mind of students by category MPK and learning models.

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Table 2 shows that, overall, the average n-gain habits of mind students in constructing a mathematical proof in the low category is 0.28. However, the average N-gain of habits of mind of the superior control class students was 0.01 from the experimental class, and each was in a low category. If evaluated based on the level of MPK, MPK high category, then the two types have increased almost balanced. N-gain of the experimental class is 0.01 of the control class and is still in the low sort with the mean of 0.20 and 0.19, respectively. Based on the MPK level, it was not much different from the MPK high category; the class experienced a nearly equal mean increase. The mean respectively 0.28 and 0.30 in the low category in the medium category. The mean N-gain control class ahead 0.02 points from the experimental class. Not much different to MPK levels are low, the achievements of the mean increase in both groups are almost the same, where the experimental class gained a mean of 0.27 (low) with a standard deviation of 0.09 whereas the average control class gain of 0.28 (low) with a standard deviation of 0.10. Descriptively, both research classes showed the average increase in the ability of habits of mind students to construct a mathematical proof is almost balanced. When viewed from the distribution of n-gain data, it can be seen that n-gain for the control class is better than the experimental class. But it is necessary to do a statistical test to find out significantly. Previously done the first test the basic assumption that normality and homogeneity test before the test statistic. Table 3 summarizes the test results of normality and homogeneity of data n-gain habits of mind students.

TABLE 3. Summary of data normality and homogeneity test n-gain habits of mind.

MPK	Learning Models	Normality	Homogeneity	Statistics test
High	DNR-M	Normal	Homogenous	*t-test*
	Conventional	Normal		
Medium	DNR-M	Normal	Homogenous	*t-test*
	Conventional	Normal		
Low	DNR-M	Normal	Non-Homogenous	*t'-test*
	Conventional	Normal		
Total	DNR-M	Normal	Homogenous	*t-test*
	Conventional	Normal		

TABLE 4. Test average difference data n-gain habits of mind.

MPK	t-test for Equality of Means	Learning models		The principle of testing
		DNR-M	PK	
High	*t*	-1.025		H_0 accepted
	df	9		
	sig. (2-tailed)	0.332		
Medium	*t*	-0.399		H_0 accepted
	df	36		
	sig. (2-tailed)	0.692		
Low	t'	1.871		H_0 accepted
	df	5.538		
	sig. (2-tailed)	0.115		
Total	*t*	-0.838		H_0 accepted
	df	58		
	sig. (2-tailed)	0.405		

Based on Table 3, only the data at the low MPK level are normally distributed but not homogeneous, so the statistical test used is the t-test. While others, the data is normal and homogeneous, the statistical test used is the t-test to see the difference in the mean. Furthermore, the equality test (mean difference) with the formulation of hypotheses

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is as follows: H0: There is no average difference between n-gain habits of mind (high, medium, low and overall) that gets the DNR-Model with students who get conventional learning. H1: There is an average difference between n-gain habits of mind (high, medium, low and overall) that gets the DNR-Model with conventional learning students. The principle of testing the hypothesis, reject H0 if the value of Sig. (2-tailed) $< \alpha = 0.05$, and in other cases H0 is accepted. Table 4 below shows the results of the analysis of the mean difference in habits of mind n-gain data based on high, medium, low, and overall MPK categories and learning models.

Based on Table 4, obtained sig (2-tailed) values > 0.05 for each MPK category and overall. So that the testing rules accept H_0. Thus, it can be concluded that there is no difference in increasing habits of mind of students in Discrete Mathematics lectures after learning DNR-Model and Conventional Learning based on MPK (high, medium, low) and overall students. The two-way ANOVA test illustrates the interaction between the learning and MPK requirements categories, and its effects on habits of mind in constructing mathematical proof are presented in Table 5.

TABLE 5. The interaction test of n-gain habits of mind, MPK, and learning model.

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	0.126[a]	5	0.025	10.483	0.000	0.493
Intercept	2.501	1	2.501	1038.539	0.000	0.951
Learning Model	0.000	1	0.000	0.052	0.820	0.001
MPK	0.118	2	0.059	24.507	0.000	0.476
Learning Model * MPK	0.008	2	0.004	1.659	0.200	0.058
Error	0.130	54	0.002			
Total	3.708	60				
Corrected Total	0.256	59				

a. R Squared = 0.493 (Adjusted R Squared = 0.446)

Two factors that affect the n-gain habits of mind students in constructing a mathematical proof is a model of learning and MPK. Based on Table 5, the factor model of learning obtained the probability (Sig) > 0.05, meaning there is no effect of the learning model to increase student habits of mind in constructing a mathematical proof. Meanwhile, the mathematical prior knowledge factor, the value of probability (Sig) < 0.05, means that prior knowledge has a significant effect on the improvement of students' habits of mind in constructing a mathematical proof. On the other hand, the interaction between the learning model with MPK is obtained by the probability (Sig) > 0.05, which means that there is no interaction effect between the learning model with MPK to improve the habits of mind of students in constructing a mathematical proof. Figure 1 shows the interaction between the learning model with MPK to increase students' habits of mind in constructing a mathematical proof.

Based on Fig. 1, it is shown that the DNR-Model is excellent for low-level MPK categories. Meanwhile, for high and medium MPK levels, it is not better than conventional learning. Overall, the best effect is the low MPK level, then the medium MPK level, and the last high MPK level. It is quite natural to change the mindset of students who have chosen prior mathematical knowledge, which is very well, which means that they already have a mature mathematical concept and perhaps even already have experience in constructing proofs. Thus, any learning model will not significantly influence the increase of habits of mind in constructing a mathematical proof.

The test results showed that there was no statistically significant difference in the increase in student habits of mind in constructing the mathematical proof between DNR-Model class with Conventional class. I was judging from the percentage of increasing habits of mind between the two classes. It is known that 100% of both DNR-Model students and Conventional class students experienced an increase in habits of mind in constructing mathematical proof with an average rate of increase of 27% and 28%, respectively. This suggests that learning DNR-Model and learning conventional give effect to a change of mind in constructing a mathematical proof. Although, the mean increase in both classes in lower categories and the test analysis results were not statistically significant.

One of the factors that cause no difference in increasing students' habits of mind in constructing mathematical proof between DNR-Model classes and conventional classes is the less time required to form the habit of thinking in constructing a mathematical proof. Lectures were in 16 sessions with a duration of 150 minutes in each meeting. Meanwhile, McGlothlin and Killen [18] suggested establishing a new habit or behavior depending on habitual behavior, individuals, and the environment, and it takes 18 days to 254 days to form a new habit. As Clear [19] said, changing a new habit requires a long process, and the most important is how to get through the process properly. It is

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changing behavior related to daily life can take from two months to eight months, how to change the habits of thought and the habit of thinking in constructing proofs. It requires patience and not only a short time.

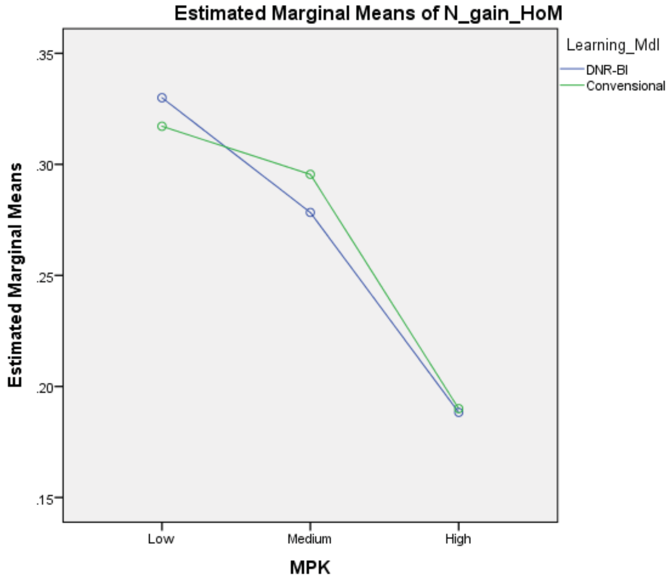

FIGURE 1. The interaction between the learning and MPK.

Another factor that led to increasing habits of mind difference is the lack of activity associated with the routine activities of daily life and the lack of activity in constructing proof repetition. This is following Dean's opinion [20], which says that a new habit will be formed if it is done through the same repetition and in the same situation, and the habits to be formed are always addressed with other routine activities in daily life.

The next factor is the regularity and continuity of the activities that form the habit of thinking in constructing proofs. The habit of thinking, in particular of constructing mathematical proof, should be done regularly and continuously. Thus, when students are faced with the problem of proof, they do not know antipathy, lazy work, or find it difficult. Because of the habits of mind in constructing the mathematical proof, they are expected to be able to think calmly, knowing what to do when faced with a problem of proof, ponder what is to be proved, to think how to find a strategy, what to do if the strategy does not work, and the other. So, the problem of proof that has been difficult to be a pleasant problem has even become a challenge for students.

Students' Habit of Mind ability is important to the attention of researchers. Because this ability supports the improvement of professional teachers [21], it is in accordance with Prasad's [22] opinion that mathematics teachers need to teach students more than just math skills, one of which is habits of mind skills. In addition, a good grading scheme also supports students in revising their mathematical thinking, taking mathematical risks, and communicating their ideas effectively through writing [22].

CONCLUSION

Habits of mind are important elements in mathematics learning activities. With the frequent exploration of students' mathematical habits of mind, students' mathematical abilities will be better, especially the ability to construct a mathematical proof. The results showed no difference in the improvement of students' habits of mind in constructing mathematical proof between the DNR-Model and conventional learning. But overall, both classes increased by 27% and 28%. Besides, the DNR-Model is very effective and significantly influences students with low-level mathematical prior knowledge. Thus, the learning model DNR-Model is very feasible to be implemented and applied in the classroom learning of mathematics. Then, changing students' habits of mind requires a long time, meaning that there needs to be ongoing and programmed attention related to students' habits of mind at every level of school education.

REFERENCES

1. A. L. Costa and B. Kallick, *Learning and Leading with Habits of Mind 16 Essential Characteristics for Students*

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(Association for Supervision and Curriculum Development, Washington D.C., 2008).
2. A. Mujib, "Analysis of student difficulties in constructing mathematical proof on discrete mathematics course," In *Proceeding of International Seminar on Mathematics, Science, and Computer Science Education* (Faculty of Mathematics and Science Education, Universitas Pendidikan Indonesia, 2015), pp. 49-55.
3. R. C. Moore, Educ. Stud. Math. **27**, 249 (1994).
4. A. Mujib, Darhim, and B.G. Kartasasmita, Int. J. Educ. Res. **4**, 287 (2016).
5. A. Mujib, "Pengembangan Kemampuan Pembuktian dalam Matematika Diskrit Menggunakan Pengajaran Berbasis DNR," in *Proceeding Seminar Nasional Matematika dan Pendidikan Matematika 2016* (FKIP Universitas Swadaya Gunung Djati, Cirebon, 2016), pp. 191–199.
6. K. Weber, Educ. Stud. Math. **48**, 101 (2001).
7. A. Hendroanto, A. Istiandaru, N. Syakrina, F. Setyawan, R. C. I. Prahmana, and A. S. E. Hidayat, Int. J. Emerg. Math. Educ. **2**, 129 (2018).
8. R. Oktaviyanthi, T. Herman, and J. A. Dahlan, J. Math. Educ. **9**, 195 (2018).
9. K. Pfeiffer, "The Role of proof validation in students' mathematical learning," in *Proceeding of the British Society for Research into Learning Mathematics* (BSRLM, London, 2009), pp. 548–550.
10. B. Schwarz and G. Kaiser, " Professional competence of future mathematics teachers on argumentation and proof and how to evaluate it," in *Proceeding ICMI Study 19 Conference Proof Proving in Mathematics Education* (The Department of Mathematics, National Taiwan Normal University, Taipei, 2009), pp. 190–205.
11. M. Shahrill, R. I. I. Putri, Zulkardi, and R. C. I. Prahmana, " Processes involved in solving mathematical problems," in *International Conference on Electrical, Electronics, Materials and Applied Science (ICEEMAS)*, AIP Conference Proceeding **1952**, 020019 (2018).
12. K. Weber, Math. Think. Learn. **12**, 306 (2010).
13. A. Selden and J. Selden, J. Res. Math. Educ. **34**, 4 (2003).
14. M.A. Nuha, S.B. Waluya, and I. Junaedi, Int. J. Instr. **11**, 527 (2018).
15. G. Harel, Am. Math. Mon. **105**, 497 (1998).
16. G. Harel, Theor. Math. Educ. 343 (2010).
17. G. Harel, "Intellectual Need," In *Vital Directions for Research in Mathematics Education*, edited by K. Leatham (Springer, New York, 2014), pp. 1-207.
18. H. McGlothlin and M. Killen, Eur. J. Soc. Psychol. Eur. **40**, 625 (2010).
19. J. Clear, HuffPost The New York Time (2014), available at https://www.huffpost.com/entry/forming-new-habits_b_5104807
20. A.S. López, Law Policy **4**, 37 (2006).
21. S. Altan, J. F. Lane, and E. Dottin, J. Teach. Educ. **70**, 169 (2019).
22. P. V. Prasad, Primus Problems Resour. Issues Math. Undergrad. Stud. **30**, 908 (2020).

Downloaded from http://pubs.aip.org/aip/acp/article-pdf/doi/10.1063/5.0096095/16224167/020038_1_online.pdf